FILE COPY

OTC 3515

PREDICTION OF THE DAMPING CONTROLLED RESPONSE OF OFFSHORE STRUCTURES TO RANDOM EXCITATION

by J.K. Vandiver, Massachusetts Institute of Technology

This paper was presented at the 11th Annual OTC in Houston, Tex., April 30-May 3, 1979. The material is subject to correction by the author. Permission to copy is restricted to an abstract of not more than 300 words.

ABSTRACT

A method is presented for predicting the damping controlled response of a structure at a known natural frequency to random wave forces. The results are applicable to a wide variety of fixed or floating structures. Potential applications include the response prediction of the fundamental flexural mode of a steel jacket structure, or the prediction of the heave, pitch, and roll responses of a tension leg platform. The principal advantage of the proposed method over those in current use is that the explicit calculation of wave forces is not required in the analysis. This is accomplished by application of the widely overlooked principle of reciprocity; that the linear wave force spectrum for the particular vibration mode is proportional to the radiation (wave making) damping of that mode. Contrary to popular belief, the conclusions show that the response to wave excitation at a natural frequency does not grow without bound as the damping is decreased, but, in fact, reaches an upper bound, which is independent of damping. Several example calculations, including the prediction of the heave response of a tension leg platform are presented. The directional distribution of the wave spectrum is included in the analysis. The examples are structured so as to be easily extended to other applications.

INTRODUCTION

This paper introduces a simple procedure for estimating the dynamic response of a structure at each of its natural frequencies to the random excitation of ocean waves. The principal advantage of the proposed method over those presently used is that the explicit calculation of wave forces has been eliminated from the analysis. This is made possible by a direct application of the reciprocity relations for ocean waves, which were established originally by Haskind[1] and described by Newman[2] in a form that is easy to implement. Briefly stated, for many structures, it is possible to derive a simple expression for the wave force spectrum in terms of the radiation damping and the prescribed wave amplitude spectrum.

The dynamic amplification of structural responses to random wave forces at a natural frequency is known to be strongly dependent on damping. This analysis shows that, for structures excited by random ocean waves, the response contributed by the damping controlled resonant band that includes the natural frequency is not governed by the total damping for that vibration mode, but, in fact, by the ratio of the radiation to the total damping.

As a consequence, knowing only the structural natural frequency of interest, the prescribed wave amplitude spectrum, and the ratio of the radiation to total damping for that vibration mode, it is relatively simple to estimate the mean square response of the structure in the frequency band that includes the natural frequency. If the ratio of the radiation to total damping is not known, an upper bound estimate of the mean square response still may be obtained.

Linear wave theory is assumed, and therefore, excitation due to drag forces is not considered. However, for many structures, drag excitation is negligible except for very large wave events. In the design process, extreme events are modeled deterministically by means of a prescribed design wave, and not stochastically as is done here. In many circumstances linear wave forces will dominate, and the results shown here will be applicable. Although drag exciting forces are not included, damping resulting from hydrodynamic drag is included. Wave diffraction effects are extremely difficult to calculate. This analysis includes diffraction effects, but never requires explicit evaluation of them.

There are numerous applications of present interest. For example, the fatigue analysis of a tension leg platform must include an estimate of the amplified responses at the natural frequencies of the structure in heave and pitch. This method quickly provides that response estimate. An example calculation for the heave response of a TLP is included. Two additional examples are provided, which exploit simplifications that frequently may be useful. The

References and illustrations at end of paper.

14. Stevens, P.M., "Instrumentation of Fixed OCS Platforms", Aero-Space Corporation Report No. ATR-77(7627-02)-1, October 1977.

15. Ruhl, J.A. and Berdahl, R., "Forced Vibration Test of a Deep Water Platform" Proceedings of the 1979 Offshore Technology Conference, Paper No. 3514, Houston, May 1979.

first is the case where the wave exciting force is independent of incidence angle, as would be true when considering the heave response of a structure with a vertical axis of symmetry. The second example illustrates the simplifications obtained when the wave spectrum is distributed broadly in incidence angle.

The method is quite general and may be applied to a variety of fixed, floating, and submerged structures that meet the criteria established in this paper. This approach highlights simply and quickly those aspects of a structural design most likely to need improvement in withstanding random wave loads; and furthermore, provides a basic tool for structural optimization.

The techniques applied in this paper are new to the field of ocean engineering. However they are not without precedent and have found extensive application in the fields of acoustics and vibration.[3]

THE LINEAR OSCILLATOR MODEL

In general, a structure in the ocean may have a large number of natural frequencies, although at only a few is the dynamic response to wave excitation likely to be important. It is convenient for the purpose of this paper to assume that by using the techniques of modal analysis each of the responding natural modes may be modeled as an independent single degree of freedom resonator. The general requirements for this are that the vibration of the structure behave in a linear fashion and that the damping be small. The motivation for using modal analysis is that it is far simpler mathematically to analyze a few independent single degree of freedom models, than one large coupled, multidegree of freedom system. Ref. 4 presents a thorough discussion of modal analysis, and Ref. 5 demonstrates its application to offshore structures.

Henceforth, this paper will be presented in terms of the response of a simple single degree of freedom resonator excited by ocean wave forces. The results should be interpreted in the larger context of modal analysis; that the total response of a structure can be obtained by a superposition of the individual responses of the modes of interest. Although it always will not be stated explicitly, the coefficients and variables of the single degree of freedom system must be expressed in terms of the appropriate modal quantities for the specific natural mode being modeled.

The equation of motion for the single degree of freedom resonator excited by ocean waves will contain terms corresponding to hydrodynamic forces as well as purely mechanical ones, such as structural stiffness. The hydrodynamic exciting forces will be, in general, a function of the relative acceleration, velocity, and displacement between the water particles and the generalized coordinates that represent the motion of the structure. For structures that behave in a linear fashion, these quantities may be expressed separately. Thus, the loads on the resonator resulting from its motion in an otherwise calm ocean may be added to the forces exerted on the resonator when held rigidly in place and loaded by the passage of ocean waves.

This may be expressed mathematically as follows, where the coefficients are often functions of frequency.

$$(m + m_a)\ddot{q} + (R_i + R_{rad} + R_v)\dot{q} + (K_s + K_{hy})q = f(\ddot{\eta}) + g(\dot{\eta}) + h(\eta), \quad \ldots \quad (1)$$

where m = modal mass of structure
m_a = modal added mass of water
R_i = linear internal structural modal damping, not related to the presence of the fluid
R_{rad} = radiation or wave making damping of the mode (a linear frequency dependent term that may be expressed by potential flow theory).
R_v = the viscous fluid modal damping (due to the assumption of light damping, it is assumed that an equivalent linearization will be adequate)
q = the appropriate normal coordinate obtained by modal analysis for this particular mode
K_s = structural modal stiffness
K_{hy} = hydrostatic modal stiffness that arises from changes in displacement of a body on the free surface

On the right side appear the excitation quantities that are functions of the water particle acceleration, velocity, and displacement $\ddot{\eta}$, $\dot{\eta}$, and η.

$g(\dot{\eta})$ = the drag force excitation term that is assumed small compared with the other two terms, and is dropped
$f(\ddot{\eta})$ and $h(\eta)$ = the hydrodynamic modal forces normally calculated from potential flow theory by integrating the pressure over the surface of the body. In fact, these are the inertial and hydrostatic forces exerted by passing waves.)

The exciting forces appearing on the right side are the modal forces that would be exerted on the body if it were held rigidly in place. A principal conclusion of this paper is that these forces need not be evaluated explicitly in order to obtain an estimate of the mean square response of a particular vibration mode.

Eq. 1 is of the form of a simple single degree of freedom oscillator.

$$M_v \ddot{q} + R_T \dot{q} + Kq = F(t), \quad \ldots \quad (2)$$

where M_v = virtual mass
R_T = total damping
K = total stiffness
$F(t)$ = modal exciting force

The undamped natural frequency and the damping ratio are given by the following familiar expressions.

$$\omega_o = \sqrt{K/M_v} \quad \ldots \quad (3)$$

$$\zeta = \frac{R_T}{2\omega_o M_v} \quad \ldots \quad (4)$$

M_V, R_T, and K generally may not be assumed independent of frequency. However, in the following analysis, the frequency range of interest is confined to a narrow band about the natural frequency. Within this band we shall assume that M_V and K do not vary. However, the frequency dependence of R_T may not be disregarded so easily. The radiation damping portion of R_T is strongly frequency dependent. Since the behavior of an oscillator at resonance is damping controlled, then the nature of the damping must be well understood before simplifying assumptions are made.

RECIPROCITY RELATIONS

In general, the evaluation of hydrodynamic forces on a body in an incident wave system is very difficult. It is necessary to know not only the hydrodynamic pressure in the incident wave system, but also the effects on this pressure field due to the presence of the body. The incident pressure field is relatively easy to evaluate, but the diffraction effects usually are extremely difficult to obtain. Haskind and Newman[1,2] have presented expressions for the exciting forces and moments on a fixed body that do not require knowledge of the diffraction effects, but depend instead on the velocity potential for forced oscillations of the body in calm water. In other words, there is a direct relationship between the radiation damping on a body that is forced to oscillate in calm water and the force exerted on that body when it is held fixed in incident waves.

Newman evaluated the expressions for an arbitrary three-dimensional body either on the surface or submerged, in terms of the six generalized coordinates and forces relating to the six rigid body degrees of freedom.

In general, one would desire the relation between the modal radiation damping coefficient and the modal exciting force. The modal exciting force and, therefore, the modal radiation damping may be obtained by a linear transformation from the six generalized forces in accordance with the method of modal analysis. It is essential to understand this relationship, but in the following analysis, one is never actually required to carry out the calculation.

The Haskind/Newman relation is stated here in terms of the modal quantities necessary in the remainder of this discussion.

$$R_{rad}(\omega) = \frac{\omega^3}{4\pi\rho g^3}\int_0^{2\pi}\frac{|F(\omega;\beta)|^2}{|A(\omega,\beta)|^2}\,d\beta\ , \quad \ldots\ldots (5)$$

where $R_{rad}(\omega)$ = radiation damping coefficient for the natural mode of interest

$F(\omega,\beta)$ = the modal exciting force exerted on the fixed body by a system of plane deep-water waves of frequency ω and amplitude $A(\omega,\beta)$, incident on the body at an angle β. [$F(\omega,\beta)$ and $A(\omega,\beta)$ both have an $e^{i\omega t}$ time-dependent term that will not be explicitly written out]

ρ = density of water

g = acceleration of gravity

This equation states that the modal radiation damping coefficient is proportional to the integral of the square of the modal exciting force, integrated over all angles of incidence.

Generally, for an arbitrary body, the wave forces will depend on the shape of the body and the angle of incidence of the waves. For this analysis it is useful to have a shape function defined as

$$\Gamma(\omega,\beta) = \frac{F(\omega,\beta)}{A(\omega,\beta)} \quad \ldots\ldots (6)$$

Γ is a measure of the modal force per unit wave height as a function of wave frequency and incidence angle. A mean square value of Γ computed over all incidence angles is given simply by

$$<|\Gamma|^2>_\beta = \frac{1}{2\pi}\int_0^{2\pi}\frac{|F(\omega,\beta)|^2}{|A(\omega,\beta)|^2}\,d\beta \quad \ldots\ldots (7)$$

Therefore, from Eq. 5, $R_{rad}(\omega)$ may be expressed in terms of the mean square value of Γ.

$$R_{rad}(\omega) = \frac{\omega^3}{2\rho g^3} <|\Gamma|^2>_\beta \quad \ldots\ldots (8)$$

Eq. 6 may be rewritten as

$$F(\omega,\beta) = A(\omega,\beta)\Gamma(\omega,\beta) \quad \ldots\ldots (9)$$

This is the modal wave force due to the incidence of regular waves of a single frequency and incidence angle. Again the time dependent $e^{i\omega t}$ term is implied and not written out explicitly. Because only linear processes are being considered, superposition of waves of many frequencies and incidence angles results in a modal wave force spectrum of the form

$$S_F(\omega,\beta) = S_\eta(\omega,\beta)|\Gamma(\omega,\beta)|^2 \quad \ldots\ldots (10)$$

When possible, the modal force spectrum may be further simplified by integrating this expression over all incidence angles.

$$S_F(\omega) = \int_0^{2\pi} S_\eta(\omega,\beta)|\Gamma(\omega,\beta)|^2\,d\beta \quad \ldots (11)$$

As will be demonstrated by example, there are many applications in which the results of this integral may be expressed in terms of the mean square of the shape function and the simple wave amplitude spectrum as shown here.

$$S_F(\omega) = C_1\, S_\eta(\omega) <|\Gamma|^2>_\beta\ , \quad \ldots\ldots (12)$$

where C_1 is a frequency independent constant that is obtained for each application and depends on the shape of the structure and the directional characteristics of the wave spectrum. How one obtains C_1 is shown in three examples at the end of the paper. From Eq. 8 the mean square value of Γ may be expressed in terms of the radiation damping. Substitution into Eq. 12 results in

$$S_F(\omega) = C_1\, S_\eta(\omega)\frac{2\rho g^3}{\omega^3} R_{rad}(\omega) \quad \ldots (13)$$

This is a result of considerable utility. The wave force spectrum has been expressed in terms of the simple wave amplitude spectrum and the radiation damping. This result leads to extremely useful expressions for the response of the resonator.

THE RESPONSE OF A SINGLE DEGREE OF FREEDOM RESONATOR TO RANDOM EXCITATION

Through the use of modal analysis, the total structural vibration has been expressed in terms of a set of independent single degree of freedom oscillators, one for each vibration mode. If the displacement of one of these oscillators is denoted by q, then the displacement response spectrum to the modal wave force spectrum $S_F(\omega)$ is given by

$$S_q(\omega) = S_F(\omega)\ |H_q(\omega)|^2 , \qquad (14)$$

where $H_q(\omega)$ is the complex frequency response of the resonator and may be found in any vibrations text.[4]

$$|H_q(\omega)|^2 = \left[\frac{1/K^2}{(1-\frac{\omega^2}{\omega_o^2})^2 + (2\zeta\frac{\omega}{\omega_o})^2}\right]. \qquad (15)$$

The modal wave force spectrum and, consequently, the modal radiation damping, $R_{rad}(\omega)$, vary with frequency. The total damping ratio ζ, through its dependence on $R_{rad}(\omega)$ is also a frequency-dependent term. The remainder of this section is devoted to presenting a simple but accurate expression for the response of the resonator that arises from the damping controlled resonant peak that is centered on the natural frequency.

From random vibration theory, the mean square of a process is given by the integral of the spectrum over all frequency. Therefore, the mean square displacement is given by

$$<q^2> = \int_0^\infty S_q(\omega)d\omega = \int_0^\infty S_F(\omega)\ |H_q(\omega)|^2 d\omega, \qquad (16)$$

where, for engineering purposes, only positive frequencies are allowed.

If the force spectrum is a constant, S_o, over all frequency, the mean square displacement is simply

$$<q^2> = S_o \int_0^\infty |H_q(\omega)|^2 d\omega. \qquad (17)$$

For light constant damping (i.e., $\zeta \leq 0.15$) the value of this integral is approximated closely by the following expression that may be found in the text by Lyon.[3]

$$<q^2> = \frac{\pi S_o}{4\zeta M^2\omega_o^3} = \frac{\pi S_o}{2R_T M\omega_o^2} , \qquad (18)$$

where $R_T = R_i + R_v + R_{rad}$, the total damping of the resonator. The largest contribution to this integral comes from the damping controlled peak in $|H_q(\omega)|^2$, which is confined to a narrow band of frequencies about the natural frequency ω_o. In fact, 64% can be attributed to the small band in frequency, $\omega_o \pm \zeta\omega_o$, known as the half power band width; $\Delta\omega = 2\zeta\omega_o$. The mean square response to S_o in the half power band may be expressed as

$$<q^2>_{\Delta\omega} = S_o \int_{\omega_o(1-\zeta)}^{\omega_o(1+\zeta)} |H(\omega)|^2 d\omega \simeq \frac{S_o}{R_T M\omega_o^2} \qquad (19)$$

$$\therefore \frac{<q^2>_{\Delta\omega}}{<q^2>} = \frac{2}{\pi} = 64\% \qquad (20)$$

If the limits of integration in Eq. 19 are doubled to include two half power band widths, $\omega_o \pm 2\zeta\omega_o$, then 80% of the total dynamic response will be included.

$$<q^2>_{2\Delta\omega} \simeq \frac{.4\pi S_o}{R_T M\omega_o^2}. \qquad (21)$$

An accurate estimate of the mean square response of a lightly damped resonator excited by ocean waves may be obtained within a half power band width. This may be done by assuming that the values of the wave force spectrum and the radiation damping at the natural frequency of the resonator, ω_o, represent acceptable averages over the band $\Delta\omega$. This assumption provides a simple but reasonably accurate estimate of the damping controlled dynamic response in the half power band, $\Delta\omega$.

$$<q^2>_{\Delta\omega} \simeq \frac{S_F(\omega_o)}{R_T(\omega_o)M\omega_o^2} \qquad (22)$$

The error introduced by this approximation is related directly to the width of the half power band $\Delta\omega = 2\zeta\omega_o$, and therefore to the total damping ζ. For very low damping ($\zeta \leq 0.05$), the error is negligible. This was confirmed by a numerical integration of Eq. 19 over the half power band for a variety of cases in which the wave force spectrum and radiation damping were allowed to vary with frequency in a realistic fashion. The worst case results indicate that the error introduced by using the approximation of Eq. 22 was less than 2% for $\zeta \leq 0.05$. This error will increase with an increase in the total damping ζ. However, for any specific application the frequency dependence of the wave force spectrum $S_F(\omega)$ and the total damping ratio ζ may be estimated in the neighborhood of the natural frequency ω_o. By evaluating the ratio between the expressions provided in Eqs. 22 and 19, the actual error may be accounted for. Such a procedure would allow the extension of the simple results of Eq. 19 to include total damping values as high as 10 or 15%.

In the case of very low total damping ($\zeta \leq 0.05$), the assumption of constant force spectrum and total damping may be increased to include a greater portion of the damping controlled peak. For example, Eq. 21 may be used to provide an estimate of the damping controlled response in a region that is two half power bandwidths wide. For $\zeta \leq 0.05$, the worst case error increases to only 6%, and approximately 80% of the total dynamic response is contained in the prediction given by

$$<q^2>_{2\Delta\omega} \simeq \frac{.4\pi\ S_F(\omega_o)}{R_T(\omega_o) M\omega_o^2} \quad \ldots\ldots\ldots\ (23)$$

To simplify the presentation in the remainder of the paper, response estimates will be made for the region defined by a single half power bandwidth using Eq. 22. It is implied that other estimates using broader bands (such as Eq. 23) also may be used, though larger errors will result.

ELIMINATION OF EXPLICIT CALCULATION OF WAVE FORCES

In an earlier section the reciprocity relation was used to derive an expression for the modal wave force spectrum in terms of the radiation damping.

$$S_F(\omega) = C_1\ S_\eta(\omega)\ \frac{2\rho g^3}{\omega^3}\ R_{rad}(\omega) . \ \ldots\ (13)$$

This expression may be substituted into Eq. 22, thereby obtaining an expression for the mean square response in the half power band, which does not require explicit calculation of the wave force spectrum.

$$<q^2>_{\Delta\omega} = \frac{2C_1\ \rho g^3\ S_\eta(\omega_o)}{M\omega_o^5}\ \frac{R_{rad}(\omega_o)}{R_T(\omega_o)} \ldots\ (24)$$

The most important feature revealed by this expression is that the damping controlled response of a resonator excited by linear ocean wave forces is dependent on the ratio of the radiation to total damping evaluated at the natural frequency, ω_o. It is far simpler to estimate the ratio $R_{rad}(\omega_o)/R_T(\omega_o)$ than it is to evaluate $R_{rad}(\omega)$ itself. Furthermore, since this ratio can never exceed 1.0, then without any knowledge of the ratio, an upper bound estimate still may be achieved. This upper bound is independent of damping. The widely held belief that the response of a structure at a natural frequency increases without bound as the damping is decreased simply is not true when the excitation is provided by linear wave forces. This is a consequence of the reciprocity relation stated in Eq. 5. It is impossible to reduce the radiation damping without also reducing the exciting forces, thus resulting in a bounded response.

The as-yet-unevaluated constant C_1 is dependent on the shape of the structure and the directionality of the wave spectrum. In the following three examples, C_1 will be evaluated. These examples were selected because they may be extended directly to a large variety of ocean structures.

Sample Response Calculations

Example 1: Heave Response of an Oceanographic Mooring

The results of this example apply to any structure for which it may be argued that the modal force is independent of wave incidence angle.

Consider the simple oceanographic mooring shown in Fig. 1. It consists of a submerged spherical float and a tripod elastic tether. The undamped natural frequency in heave is given by Eq. 3, where K is a linear stiffness coefficient for small vertical motions.

$$\omega_o = \sqrt{K/M_v} \ \ldots\ldots\ldots\ldots\ (3)$$

The modal force for vibration in the vertical direction is simply the generalized force in the vertical direction on the float. Furthermore, because the float has a vertical axis of symmetry, the heave exciting force is independent of the angle of incidence of the waves, and therefore $\Gamma(\omega,\beta)$ is a function of ω only.

From Eq. 11, the modal exciting force is

$$S_F(\omega) = \int_0^{2\pi} S_\eta(\omega,\beta)\,|\Gamma(\omega,\beta)|^2 d\beta\ .\ \ldots(11)$$

Since Γ is independent of β, it may be moved outside of the integral. We may note further that in this case the magnitude squared of Γ and its mean square with respect to β must be equal.

$$|\Gamma|^2 = <|\Gamma|^2>_\beta . \ \ldots\ldots\ldots\ldots (25)$$

Therefore,

$$S_F(\omega) = <|\Gamma|^2>_\beta \int_0^{2\pi} S_\eta(\omega,\beta)\,d\beta\ \ldots\ (26)$$

$$= <|\Gamma|^2>_\beta\ S_\eta(\omega) \ \ldots\ldots\ldots (27)$$

because the integration of the directional wave spectrum over all incidence angles results in the simple wave amplitude spectrum.

This result is now in the form of Eq. 12 when $C_1 = 1.0$. It follows that we may express the heave exciting force spectrum in the form of Eq. 13, with $C_1 = 1.0$.

$$S_F(\omega) = S_\eta(\omega)\ \frac{2\rho g^3}{\omega^3}\ R_{rad}(\omega)\ ,\ \ldots\ (28)$$

where $R_{rad}(\omega)$ is now the modal radiation damping of the spherical float for heave motions.

The heave response spectrum is the same as presented in Eq. 14,

$$S_q(\omega) = S_F(\omega)\ |H_q(\omega)|^2 \ \ldots\ldots\ldots (14)$$

and the mean square response in the small half power band about the natural frequency is from Eq. 24,

$$<q^2>_{\Delta\omega} = \frac{2\rho g^3}{M\omega_o^5}\ S_\eta(\omega_o)\ \times\ \frac{R_{rad}(\omega_o)}{R_T(\omega_o)} .\ \ldots(29)$$

This estimate of the heave response of the buoy is appropriate within the half power band $\Delta\omega = 2\zeta\omega_o$, provided the system is reasonably linear, the total damping is small, and the assumptions and limitations of modal analysis are satisfied.

In this prediction of the heave response of a mooring, no mention was made of the dependence on the depth of submergence. This is implicit in the ratio

$R_{rad}(\omega_0)/R_T(\omega_0)$. Newman shows that the radiation damping coefficient decreases as e^{-2kh}, where k is the wave number of radiated waves. In the limit that the depth of submergence $h \to \infty$, then $R_{rad}(\omega_0) \to 0$ and the ratio also goes to zero. Thus the response of the buoy is predicted correctly to be zero at depths below the region of significant wave excitation.

The specific results shown in Eqs. 28 and 29 for this example generally are applicable to a broad range of structures; that is, whenever the modal exciting force is independent of wave incidence angle. As will be shown in the next section, these results also apply whenever the waves in the frequency band of interest can be assumed to have random incidence angle.

Example 2: Random Incidence Waves

When the incident wave spectrum is distributed equally over all incidence angles, the results shown in Eqs. 28 and 29 apply. This is relatively easy to demonstrate, even for structures with complicated or unknown shape functions. For waves of completely random incidence angle, the directional wave spectrum and the simple amplitude spectrum are related in the following way,

$$S_\eta(\omega,\beta) = \frac{1}{2\pi}\, S_\eta(\omega)\,. \quad \ldots\ldots\ldots \quad (30)$$

This may be substituted into Eq. 11, the general expression for the force spectrum.

$$S_F(\omega) = \int_0^{2\pi} S_\eta(\omega,\beta)\ |\Gamma(\omega,\beta)|^2 d\beta$$

$$= S_\eta(\omega)\ \frac{1}{2\pi}\int_0^{2\pi} |\Gamma(\omega,\beta)|^2 d\beta, \ldots \quad (31)$$

where the angular independent wave spectrum has been moved outside of the integral. The integral now is reduced to that which defines the mean square of Γ with respect to β. Therefore,

$$S_F(\omega) = S_\eta(\omega)\ \langle|\Gamma|^2\rangle_\beta\,, \ldots\ldots\ldots (27)$$

which, when expressed in terms of $R_{rad}(\omega)$, yields the same results as the case in which the exciting force was independent of β.

$$S_F(\omega) = S_\eta(\omega)\ \frac{2\rho g^3}{\omega^3}\ R_{rad}(\omega) \quad \ldots \quad (28)$$

Of course, this immediately leads to the same expression for the response in the half power band width as found in Eq. 29 of the previous example. In fact, for the result shown in Eq. 29 to be valid, it is necessary only that the waves whose frequency lies within the half power band be randomly incident. Waves outside of the band need not be so randomly oriented. As a practical matter the high frequency components of a seaway tend to be more confused in direction than the low frequency waves. Therefore, the validity of the assumption of randomly incident waves may be more appropriate than ordinarily supposed, depending on the natural frequency of the structure, geographic location, and prevailing weather conditions.

The result just shown applies to an arbitrary shape function. Any structural symmetries will reduce the range of angles over which the waves must be randomly incident. For example, it can be shown that for a structure with two orthogonal vertical planes of symmetry, such as a steel jacket platform with a rectangular layout of its primary legs, the waves in the half power band need be only randomly incident over a semicircle, i.e., 180° for Eqs. 27, 28, and 29 to hold. The result might be used to predict the mean square response of the two lowest flexural modes.

For many structures these simplifying assumptions may be justified, and the simple result for the mean square response within the half power band width as shown in Eq. 29 may be applied.

However, at times such assumptions may not be acceptable, and it may be necessary to measure or estimate $\Gamma(\omega,\beta)$ and also to incorporate a directional wave spectrum $S_\eta(\omega,\beta)$. Such a procedure is followed in the final example.

Example 3: The Response of a Tension Leg Platform to Random Wave Excitation

An important concern in contemporary design of all platforms is fatigue. The prediction of the fatigue life is a process that must include the anticipated wave statistics and response statistics of the structure. Numerous authors have reported on difficulties encountered in estimating the response at the resonant frequencies of the structure and have noted that the response prediction for the frequency band about resonance is critically dependent on the value of damping that is selected. The method presented here is directed specifically at predicting the response in the resonant band and, beyond that, puts the role of damping in the proper perspective. Knowledge of the total damping is not sufficient. It is important to know the way in which the damping is distributed between radiation and all other sources.

Consider the hypothetical square tension leg platform shown in Fig. 2. At the preliminary design stage it would be useful to have an estimate of the response of the structure to a prescribed sea state at its natural frequencies in heave, pitch, and roll. In the following example only the heave response will be estimated. The response in the roll and pitch modes would be carried out in a very similar fashion. The primary purpose of this example is to illustrate the method one might use to take the geometry of the structure and the directionality of the wave spectrum into consideration.

The influence of both the directionality of the wave spectrum and the geometry of the structure has been compressed into the unknown constant C_1, which appears in Eq. 24, the prediction of the mean square displacement response in the half power band width.

$$\langle q^2\rangle_{\Delta\omega} = \frac{2C_1\ \rho g^3\ S_\eta(\omega_o)}{M\omega_o^5}\ \frac{R_{rad}(\omega_o)}{R_T(\omega_o)} \ldots (24)$$

A useful expression for C_1 may be obtained by eliminating the common force spectrum term, $S_F(\omega)$, from Eqs. 11 and 12 and solving for C_1. The result is shown in Eq. 32, where the integral form of the mean square of $\Gamma(\omega,\beta)$ has been used to replace the < > notation,

$$C_1 = \frac{\int_0^{2\pi} S_\eta(\omega,\beta) \; |\Gamma(\omega,\beta)|^2 d\beta}{S_\eta(\omega) \frac{1}{2\pi} \int_0^{2\pi} |\Gamma(\omega,\beta)|^2 d\beta} \quad . \; . \; . \; (32)$$

The directional wave spectrum is prescribed and in this example is assumed to be a cosine squared distribution about some reference angle β_o.

$$S_\eta(\omega,\beta) = \frac{2}{\pi} S_\eta(\omega) \cos^2 (\beta-\beta_o) , \; . \; . \; (33)$$

which is valid for $-\pi/2 \leq \beta - \beta_o \leq \pi/2$ and zero elsewhere. It is noted that

$$S_\eta(\omega) = \int_{\beta_o - \pi/2}^{\beta_o + \pi/2} S_\eta(\omega,\beta) \; d\beta \; . \; . \; . \; . \; (34)$$

By substituting into Eq. 32 the expression for $S_\eta(\omega,\beta)$ and noting that the common term $S_\eta(\omega)$ cancels out, the following is obtained.

$$C_1 = \frac{\int_{\beta_o - \pi/2}^{\beta_o + \pi/2} \frac{2}{\pi} \cos^2 (\beta-\beta_o) \; |\Gamma(\omega,\beta)|^2 d\beta}{\frac{1}{2\pi} \int_0^{2\pi} |\Gamma(\omega,\beta)|^2 \; d\beta} ,$$

$$. \; (35)$$

As generally will be the case, the problem has reduced to the need for an estimate of the angular dependence of $|\Gamma(\omega,\beta)|$. This task is simplified by the observation that an expression valid for all frequencies, ω, is not necessary. An estimate valid at only the natural frequency of interest, ω_o, is sufficient. In Fig. 3, plane progressive deep-water waves of unit amplitude and frequency, ω_o, are shown approaching the TLP at an angle β. The magnitude of the heave force exerted on a single axially symmetric leg is independent of incidence angle and may be expressed as $|\Gamma_o(\omega)|$.

The magnitude of the force exerted on the entire structure will depend primarily on the relative phases of the four individual leg forces and upon any diffraction effects. The diffraction effects are assumed small compared with the phase effects and are ignored. The magnitude of the total heave force accounting for phase effects is given by

$$|\Gamma(\omega_o,\beta)| =$$

$$4 \; |\Gamma_o(\omega_o)| \; \cos \; \left(\frac{\pi d}{\lambda} \cos\beta\right) \; \cos \; \left(\frac{\pi d}{\lambda} \sin\beta\right) ,$$

$$. \; (36)$$

where d is the leg spacing and λ is the wave length that corresponds to a frequency ω_o. Substitution of this expression into Eq. 35 yields for C_1 the result

$$C_1 = \frac{\int_{\beta_o - \pi/2}^{\beta_o + \pi/2} \cos^2(\beta - \beta_o) \cos^2(\frac{\pi d}{\lambda} \cos\beta) \cos^2(\frac{\pi d}{\lambda} \sin\beta) \; d\beta}{\int_0^{2\pi} \cos^2(\frac{\pi d}{\lambda} \cos\beta) \cos^2(\frac{\pi d}{\lambda} \sin\beta) d\beta} .$$

$$. \; (37)$$

This expression was integrated numerically for all combinations of heave natural period and leg spacing ranging from 1 to 4s and 100 to 300 ft. To 0.1% accuracy, $C_1 = 1.0$ for all directions of incidence, β_o, of the cosine squared wave spectrum. The cosine squared distribution was sufficiently broad to smooth out the effects of varying wave force phases on the four legs. This unexpected but simple conclusion allows the use of the simple result of the previous two examples. The mean square heave response in the damping controlled half power band is given by

$$<q^2>_{\Delta\omega} = \frac{2\rho g^3 \; S_\eta(\omega_o)}{M\omega_o^{\;5}} \; \frac{R_{rad} \; (\omega_o)}{R_T(\omega_o)} .$$

$$. \; (29)$$

For the large legs of a TLP, the radiation damping likely will be the greatest contributor to the total damping. Consequently, a conservative but reasonable upper bound estimate for the ratio of the radiation to total damping is 1.0, and Eq. 29 reduces to

$$<q^2>_{\Delta\omega} = \frac{2\rho g^3 \; S_\eta(\omega_o)}{M\omega_o^{\;5}} \quad . \; . \; . \; . \; . \; . \; . \; . \; (38)$$

An example calculation where

$M = 20{,}000 \;\; Mg$: the virtual mass of the TLP in heave,

$\omega_o = 2.1$ radians/sec: corresponds to a heave period of three seconds,

$S_\eta(\omega_o) = 1.89 \times 10^{-2} \; m^2 - sec$: calculated for a 15 m/s Pierson-Moskowitz spectrum,

yields a root mean square heave amplitude of

$$\sqrt{<q^2>_{\Delta\omega}} = 5.1 \; mm \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (39)$$

The heave response is insignificant. However, to arrive at that conclusion by any other means would have been much more difficult.

CONCLUSIONS

A method has been presented for predicting the damping controlled dynamic response of an offshore structure. The method is applicable to a wide variety of structures and depends only on the assumptions of linearity of wave forces and structural response and, furthermore, requires that the total structural damping be small.

There are three principal conclusions to be drawn. First, the linear wave force spectrum on a structure may be expressed in terms of the radiation damping of the structure. This is a consequence of the principle of reciprocity for ocean wave forces that has been known for many years, but has not been applied to common ocean engineering problems.

Second, through the use of the above result, a method for estimating the damping controlled response of a structural natural mode has been presented that does not require explicit calculation of the modal wave force spectrum.

Last, the role of damping in the estimation of dynamic response is placed in the proper perspective. It is not the total damping of a vibration mode that governs the response to wave excitation but, in fact, the ratio of the radiation to total damping. Since this ratio has an upper bound of 1.0, then the response has an upper bound independent of the exact value of the damping.

The conclusions of this paper are a consequence of the often overlooked principle of reciprocity between exciting forces and radiation damping.

NOMENCLATURE

$A(\omega,\beta)$ = plane progressive waves of amplitude A, frequency ω, and incidence angle β
C_1 = constant dependent on $S_\eta(\omega,\beta)$ and $\Gamma(\omega,\beta)$
d = leg spacing on TLP
$F(t)$ = modal wave force on fixed structure
$F(\omega,\beta)$ = modal wave force on fixed structure due to waves A (ω,β)
$f(\ddot{\eta}) + h(\eta)$ = linear wave forces on fixed body
$g(\dot{\eta})$ = drag exciting force on fixed body
g = acceleration of gravity
$H_q(\omega)$ = frequency response of linear second-order single degree of freedom system
K_s, K_{hy} = structural and hydrostatic contributions to the modal stiffness
K = total modal stiffness
m = modal structural mass
m_a = modal added mass
M_v = total modal virtual mass
q = modal displacement coordinate
$\langle q^2\rangle_{\Delta\omega}$ = mean square displacement in the band $\Delta\omega$
R_i = linear nonhydrodynamic damping
R_{rad}, $R_{rad}(\omega)$ = linear radiation damping
R_v = linearized viscous hydrodynamic damping
R_T = total linearized damping
$S_F(\omega,\beta)$ = directional modal wave force spectrum
S_o = constant force spectrum
$S_\eta(\omega)$ = wave amplitude spectrum
$S_\eta(\omega,\beta)$ = directional wave spectrum
$S_q(\omega)$ = modal displacement spectrum
β = wave incidence angle
$\Gamma(\omega,\beta)$ = modal wave force $F(\omega,\beta)$ per unit wave amplitude
$\langle|\Gamma|^2\rangle_\beta$ = mean square of $\Gamma(\omega,\beta)$ with respect to β
ζ = total modal damping ratio
$\eta, \dot{\eta}, \ddot{\eta}$ = water partial displacement, velocity, and acceleration
λ = wave length with frequency ω_o
ρ = density of water
ω = wave frequency
ω_o = natural frequency of the mode
$\Delta\omega$ = half power band width
$|\ |$ = denotes magnitude of

ACKNOWLEDGMENTS

This research was sponsored by grants from the National Science Foundation and the Henry L. Doherty Foundation. This research is continuing with the sponsorship of the Branch of Marine Oil and Gas Operations of the USGS. Particular acknowledgment is extended to Walter E. Schott III and Bruce Dunwoody, whose technical contributions were invaluable in the preparation of this paper.

REFERENCES

1. Haskind, M. D.: "The Exciting Forces and Wetting of Ships in Waves," (in Russian) Izvestia Akademii Nauk S.S.S.R., Otdelenie Tekhnicheskikh Nauk (1957) No. 7, 65-79. (English translation available as David Taylor Model Basin Translation No. 307, March 1962).
2. Newman, J. N.: "The Exciting Forces on Fixed Bodies in Waves," J. Ship Research, SNAME (Dec. 1962) 6, No. 3.
3. Lyon, R. H.: Statistical Energy Analysis of Dynamical Systems: Theory and Applications, MIT Press (1975).
4. Meirovitch, L.: Elements of Vibration Analysis, McGraw-Hill Book Co., Inc., New York City (1975).
5. Berge, B. and Penzien, J.: "Three-Dimensional Stochastic Response of Offshore Towers to Wave Forces," paper OTC 2050, Proc. of Sixth Annual Offshore Technology Conference, Houston, May 6-8, 1974, 173-190.

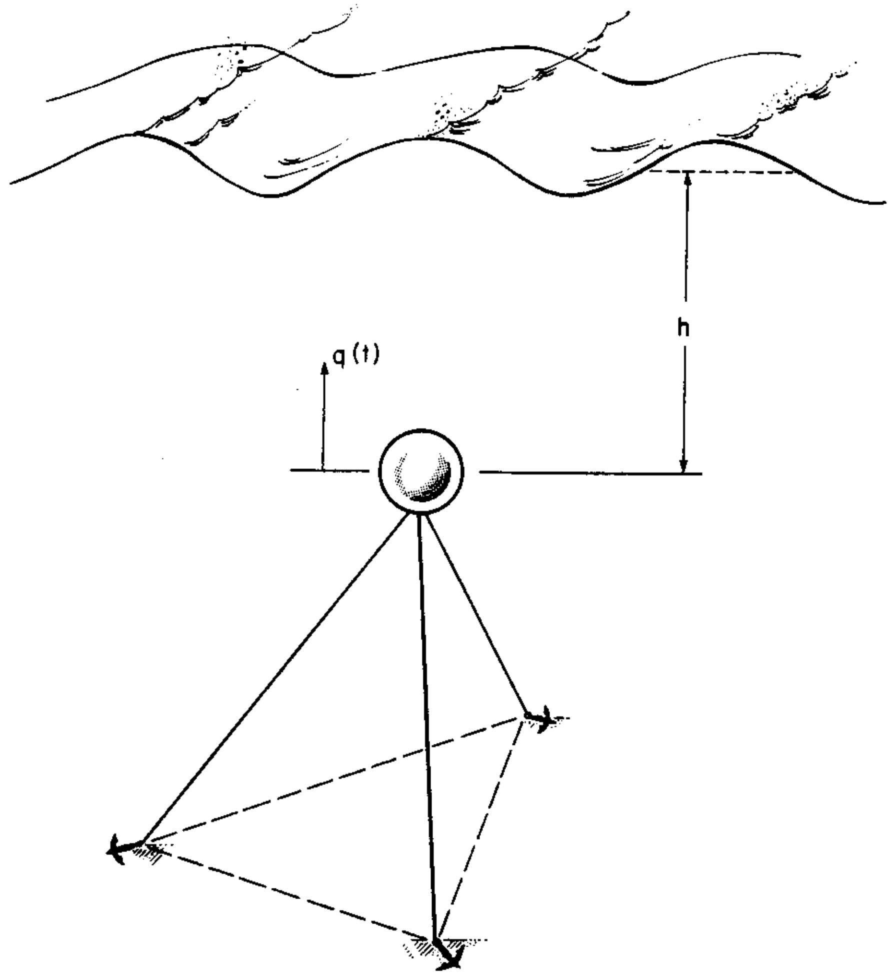

FIG. 1 - OCEANOGRAPHIC MOORING.

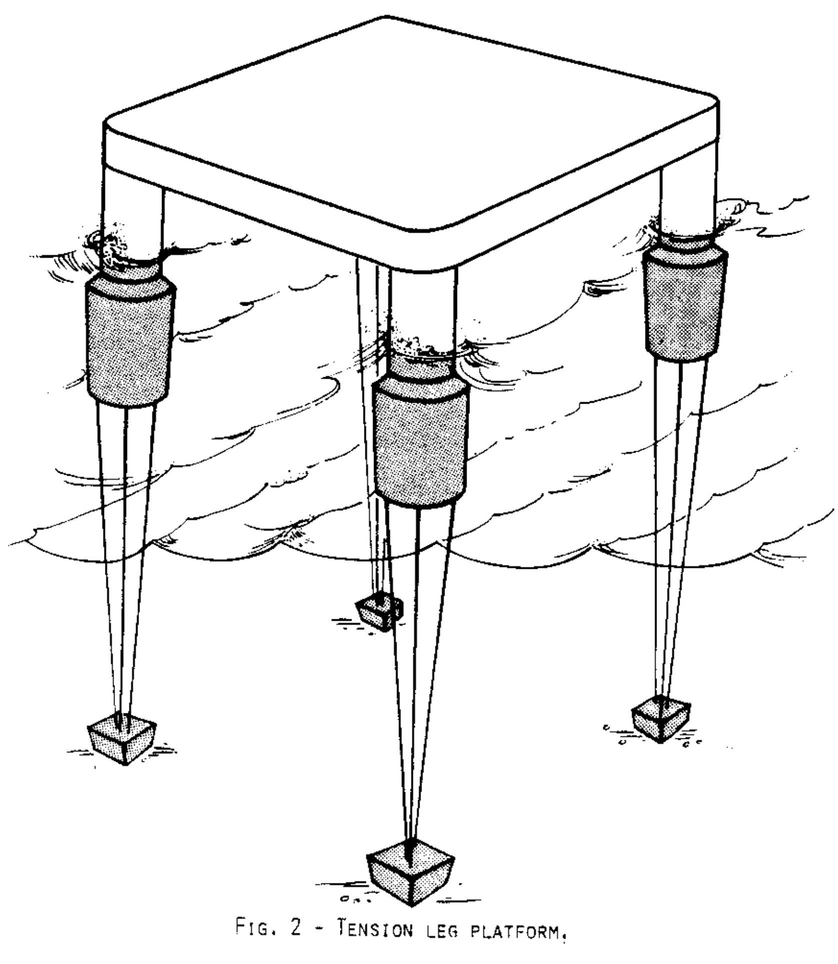
FIG. 2 - TENSION LEG PLATFORM.

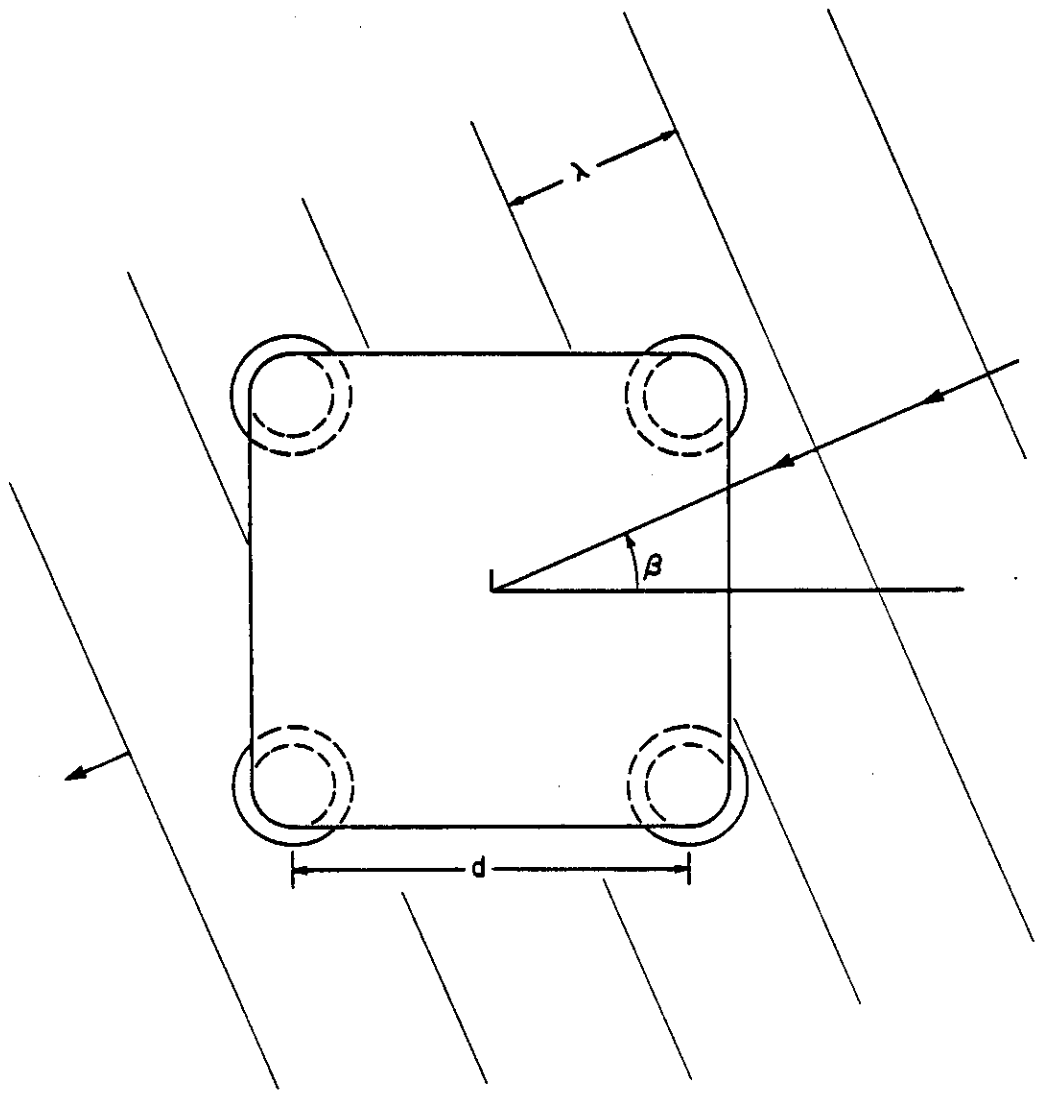

Fig. 3 - Regular waves incident on the TLP (top view).